Alongshore Propagating Waves in the Nearshore Region

Uday Putrevu
Northwest Research Associates
14508 NE 20th Street
Bellevue, WA 98007
phone: (425) 644 9660 ext. 331 fax: (425) 644 8422 email: putrevu@nwra.com

James T. Kirby
Center for Applied Coastal Research
University of Delaware
Newark, DE 19716
phone: (302) 831 2438 fax: (302) 831 1228 email: kirby@udel.edu

Award #: N00014-00-M-0017 and N00014-00-1-0076
http://chinacat.coastal.udel.edu/~kirby

LONG-TERM GOAL

Our long-term goal is to develop a model that is capable of predicting the low-frequency wave climate on open coastal beaches given the offshore wind-wave climate and the underlying bathymetry.

OBJECTIVE

The objective of this project is to investigate how the various low-frequency, alongshore-propagating waves are relatated to each other and to determine how they interact with one another. This objective constitutes the first step towards our long-term goal.

APPROACH

We derived a general formalism to compute the nonlinear interactions between coastally-trapped gravity and vorticity waves. To do so, we developed a spectral model describing the nonlinear interactions between the free waves of the system by means of resonant interactions at second order. To date the literature has identified the possibility of such resonances for the case of three edge waves (Kenyon, 1970; Bowen, 1976) or three shear waves (Shrira et al., 1997). To this list we add the possibility of triads involving a single shear wave and two edge waves as illustrated in Figure 1.

WORK COMPLETED

We have completed the following tasks:

1) We derived the general equations that govern the eigenstructure of the free modes.
2) We derived the general equations that govern the nonlinear interactions among the various modes.
3) We developed numerical solutions to determine the eigenvalues and eigenfunctions of the free modes over arbitrary bathymetries and longshore currents.

4) We derived the general form of the triad interaction coefficients.
5) We are in the process of preparing a research paper (Kirby et al.) which reports on the work carried out under this project.

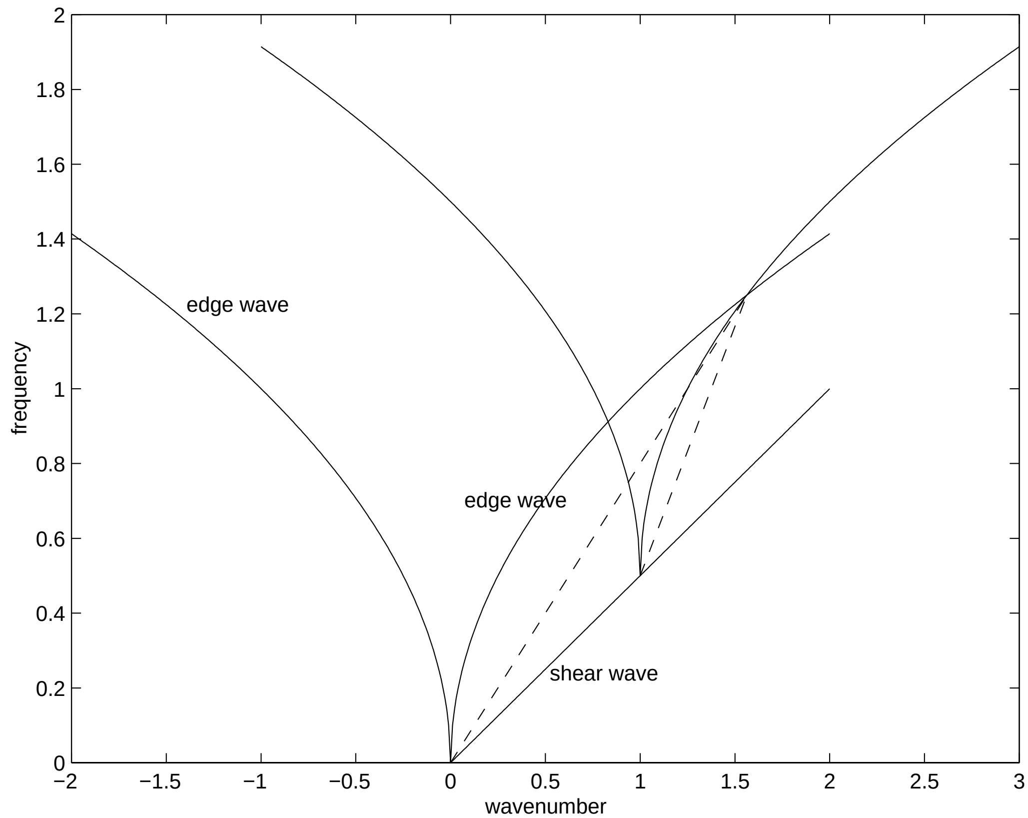

Figure 1: Resonant triad interaction involving two edge waves and one shear wave. The two dashed lines indicate two edge waves with the same mode number and propagating in the same direction as the shear wave that they are in resonance with. The second intersection point (not labelled) represents the resonance involving two counter-propagating edge waves and one shear wave.

RESULTS

We demonstrated that edge and shear waves are members of the same non-Sturm-Liouville eigenvalue problem and that they have several features in common. For example, they satisfy a common orthogonality condition, have the same general expressions for phase speed and group velocity, etc. As mentioned earlier, we derived the general expressions (in terms of the eigenstructure) for the interactions coefficients—we investigated the nonlinear energy transfer among the various modes by using these general expressions and either analytical [for edge waves on planar beaches and over Ball's (1967) profile] or numerical solutions for the eigenstructure. The results are discussed below.

We discuss the case of interacting edge wave triads first. We found that the time scale of the energy transfer (and hence the importance of the nonlinear interactions) is critically dependent on the steepness of the bathymetry. For relatively steep beaches, like those that are typical of pocket beaches we have found that the time scale of the interaction is fairly short. For example, on a beach with a slope of 1/10 the time scale for the energy transfer is of order 10 wave periods (Figure 2). This time scale is similar to the time scale over which the edge waves are generated due to direct forcing by modulated incident waves (Lippmann et al., 1997). Hence the nonlinear energy transfers will play an important role in the development of the low-frequency edge wave climate on steep beaches.

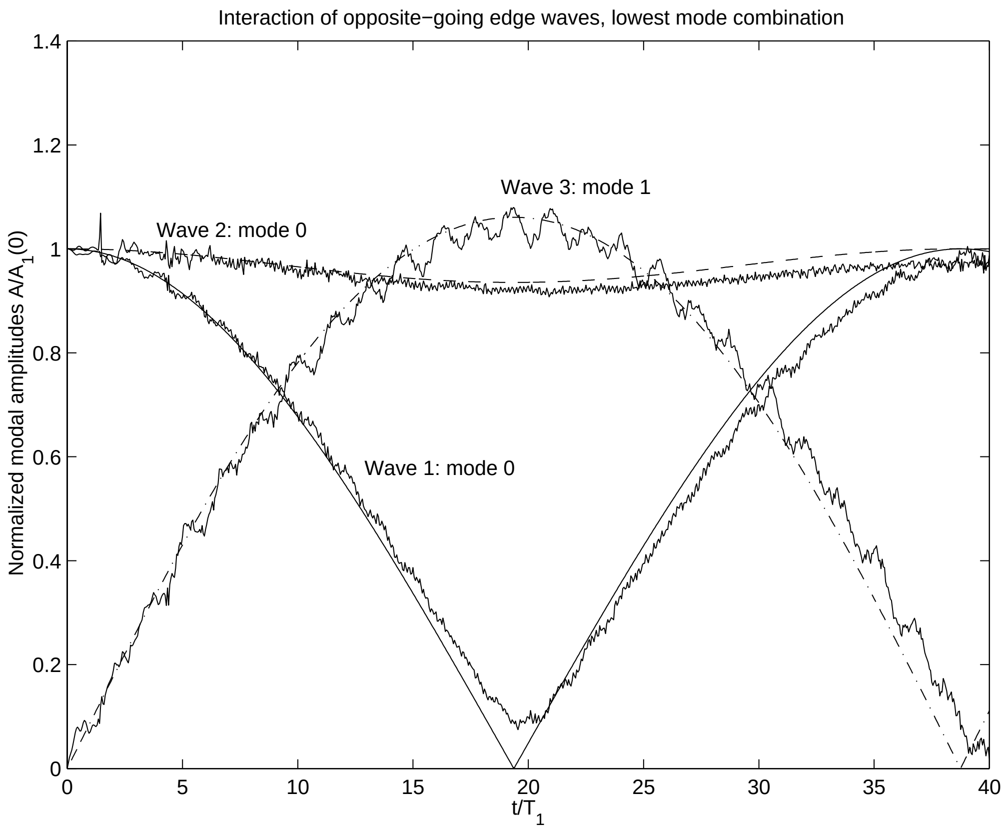

Figure 2: The temporal evolution of modal amplitudes in an edge wave triad on a planar beach (slope 1/10). The smooth lines represent the solution from the spectral model developed here and the jagged lines represent the predictions of a fully numerical calculation using the nonlinear shallow-water equation solver of Özkan-Haller and Kirby (1997). The three edge waves participating in the triad are: Wave 1—mode 0, frequency ω_1, wavenumber λ_1; Wave 2—mode 0, frequency $\omega_1/2$, wavenumber $-\lambda_1/4$; Wave 3—mode , frequency $3\omega_1/2$, wavenumber $3\lambda_1/4$. ω_1 corresponds to a period (T_1) of 20 s.

For flatter beaches (like those that may occur in typical open coastal situations) the time scale over which the nonlinear energy transfer takes place is much longer—the time scale turns out to be inversely proportional to the square of the beach slope. Hence, for flat beaches, the nonlinear energy transfer probably plays an insignificant role in the development of the low-frequency edge wave field. Furthermore, the nonstationarity of wave climate over these time scales would make the detection of the effects of the interaction very difficult, if not impossible, for such beaches.

We came across a curious result that involves interacting edge wave triads on planar beaches in the absense of longshore currents—cases involving colinear waves have zero interaction coefficients indicating a total lack of interaction. We have confirmed this result by direct numerical simulation but have not found the reason for the non-interaction. However, this result does not carry over to more general bathymetries and for that reason we did not pursue this matter further.

The characteristics of the results involving the interaction of one shear wave and two edge waves are similar to those that involve edge waves alone. The time scales of the interactions lengthen with decreasing beach slope and hence such interactions are probably unimportant on all but very steep beaches.

We were particularly interested in the interaction of a shear wave and two edge waves on planar beaches. Our interest in this problem stemmed from the fact that stability calculations on the depth profile at Leadbetter Beach (which is planar) show that the longshore currents are expected to be stable over such profiles (Dodd et al., 1992). Hence one would not expect to see shear waves over such topographies. However, the field data clearly shows the presence of shear waves. A completely satisfactory explanation to the observations of shear waves at Leadbetter beach has not been given so far. We wondered whether energy could be transferred into shear wave modes by their interaction with two edge wave modes and thereby explain the shear wave observations. We found that pathways for such energy transfers exist but that the time scale is so large that it makes such an energy transfer extremely inefficient. This result was somewhat disappointing and the search for a convincing explanation for shear wave observations on planar beaches continues.

IMPACT/APPLICATION

The finding that the time scales for nonlinear energy transfer are small on steep beaches indicates that the low-frequency wave climate on steep beaches cannot be predicted without accounting for such energy transfers. The finding that the time scales are lengthy for flat beaches suggests that we may be able to ignore the nonlinear energy transfers while predicting the low-frequency wave climate on open coastal beaches.

TRANSITIONS

None.

RELATED PROJECTS

None.

REFERENCES

Bowen, A.J., 1976. Wave-wave interactions near the shore. *Waves on Water of Variable Depth*, D.G. Provis and R. Radok (eds), Lecture notes in Physics, **64**, 102-113.

Dodd, N., J. Oltman-Shay, and E.B. Thornton, 1992. Shear instabilities in the longshore current: A comparison of observation and theory. *J. Phys. Oceangr.*, **22**, 62-82.

Kenyon, K.E., 1970. A note on conservative edge wave interactions. *Deep Sea Res.*, **17**, 197-201.

Lippmann, T.C., R.A. Holman, and A.J. Bowen, 1997. Generation of edge waves in shallow water. *J. Geophys Res.*, **102**, 8663-8679.

Özkan-Haller, H.T., and J.T. Kirby, 1997. A Fourier-Chebyshev collocation method for the shallow water equations including shoreline runup. *Applied Ocean Res.*, **19**, 21-34.

Shrira, V.I., V.V. Voronovich, and N.G. Kozhelupova, 1997. Explosive instabilites of vorticity waves. *J. Phys. Oceanogr.*, **27**, 542-554.

PUBLICATIONS

Kirby, J.T., U. Putrevu, and H.T. Özkan-Haller, 1998. Evolution equations for edge waves and shear waves on longshore uniform beaches. *Proc. 26th Intl. Conf. Coast. Engrg.,* Copenhagen, 203-216.

Kirby, J.T., U. Putrevu, and H.T. Özkan-Haller, 2000. Edge and shear wave interactions? *AGU Fall Meeting*, Abstract.

Kirby, J.T., U. Putrevu, and H.T. Özkan-Haller, in preparation. A spectral model for energy transfer among low-frequency waves in the nearshore. To be submitted to *J. Fluid Mech*.